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A CASUALTY STRATIFICATION
MODEL

by

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A Casualty Stratification Model

by

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ABSTRACT

This Naval Postgraduate School thesis develops an algorithm and model for stratifying battle casualties. The algorithm was constructed in response to a need by Headquarters Marine Corps for casualty estimates stratified by military occupational specialty and rank. The model focuses on three factors which are considered to be fundamental to an individual's survival on the battlefield: location on the battlefield, the firing rates of the enemy's weapons, and an individual's vulnerability to the enemy's weapons. A brief outline of how the model can be enhanced for added realism and usefulness is also provided.

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I. INTRODUCTION

The United States has been recognized as the leader of the free world for many years. The responsibilities of that leadership and the global nature of the nation's other strategic and defense interests require the United States Armed Forces to be prepared for a wide variety of military actions. Planning is an essential element of the preparation, and military planners continually develop, update, and revise plans for many contingencies. Imbedded within these plans are troop replacement, medical, and wartime training requirements, to name a few. Should a particular contingency arise, actual requirements such as these are dependent upon how many casualties will occur during the conflict. The number of casualties that will occur is, of course, unknown and casualties must be estimated. Thus, casualty estimation is vital to the proper development of contingency plans.

Estimating casualties for a contingency plan is one of the most difficult problems faced by wartime planners. This is so for many reasons, but only a few will be mentioned here. First the scenario must be defined. Planners must determine, for instance, the number and type of enemy and friendly forces to be employed and how that employment will occur, the effectiveness of the enemy's weapons, the training level and morale of the enemy troops, the potential for nuclear, biological, and chemical warfare, and so on. Most scenario elements concerning the opposing forces are provided by intelligence reports and estimates, but some are determined by plain old 'crystal balling'. After the scenario has been well defined, the casualties must be estimated, and two possible sources of casualty estimates are data bases from previous conflicts and experimentation. However, due to the advances in weaponry since Viet Nam, casualty data that exist from past combat experiences are not suitable for predicting casualties in future hostilities, and collecting casualty data experimentally is neither possible nor practicable. Indeed casualty estimation is a difficult problem and, in general, the military has turned to computer simulations for a solution.

A computer simulation used for casualty prediction can usually be classified as one of two types. It can be either a high resolution model or an aggregate model. An aggregate model determines losses based upon some measure of the combat effectiveness of an entire unit against its opponent. A high resolution model is much

more detailed, often taking into account the effectiveness of each weapon system on the opposing force, weather and terrain factors, individual unit movements, and so forth. Each type of model has advantages and disadvantages when compared to the other. An aggregate model is relatively easy to construct and maintain, but the result is an aggregate casualty estimate based upon force-on-force comparisons. Conversely, a high resolution model may result in detailed casualty estimates for subsets of a larger military force. However, a high resolution model is costly to build, maintain, and operate.

Headquarters, United States Marine Corps (HQMC) requires casualty estimates for planning purposes. In the past, estimates were determined for HQMC planners by aggregate models resulting in aggregate casualty estimates. That is, for a specified scenario only *total* casualties were estimated. Recently the need for more detailed manpower planning was recognized and, therefore, aggregate casualty estimates are no longer sufficient. The more detailed planning requires casualty estimates for each military occupational specialty (MOS) and rank, which we shall call casualty stratification.

The objective of this thesis is an algorithm that will provide, for a given scenario, casualty estimates by military occupational specialty and rank. We begin our discussion in the next chapter by considering what is known about the problem and some assumptions that are required for our casualty stratification algorithm. In the following chapter we discuss the necessary probabilistic concepts, and Chapter IV provides a completely worked example. Chapter V concludes this thesis with a discussion of how the model can be enhanced for added realism and usefulness.

II. STRUCTURING THE PROBLEM

A. THE BASIC ASSUMPTIONS

The first step in developing our casualty stratification algorithm will be to discuss what is known, or assumed to be known, about the problem. Suppose we are interested in stratifying the casualties for a particular contingency plan. As mentioned in the introduction, a casualty estimate pertains to a specific scenario. Therefore, we will assume that the scenario is well defined and that, for some time period, the following are known:

- initial strengths of the friendly and enemy forces,
- number, type, and maximum effective ranges of the weapons to be employed by the enemy forces,
- the approximate locations of the enemy's weapons,
- the width and depth of the battlefield,
- the intensity of combat as measured by the firing rates of the enemy's weapons,
- how each military occupational specialty will be deployed within the area occupied by the friendly forces.

We assert here that the initial strength of the friendly forces is known. This means the initial strength of each military occupational specialty is known, and within that, we know the military rank structure of each MOS.

A casualty estimate would be meaningless if it were not related to the time in combat for which the estimate was made. Suppose that for our scenario we were given an estimate of 276 casualties. Probably the first question that one might ask would be something like: Is that the number of casualties we can expect during the first day, week, or month, or is it for the second and third months combined, or . . . ? Obviously an estimate becomes significant only with the perspective of time. Thus, when we refer to a casualty estimate, we will be making the implicit assumption that we know the length of time for which the estimate applies. *Time in conflict*, and *time period* will be used interchangeably, but both have the same meaning as the length of time for which a casualty estimate applies. When we refer to initial or terminal strengths, we mean the strengths at the beginning or end, respectively, of this time period.

We will make three assumptions concerning the weapons being fired into the friendly force's area. These assumptions are that:

- weapon positions remain constant during the time period,

- all of the enemy's fire is unaimed during the time period, and
- all weapon impacts are independent and uniformly distributed within known areas.

By unaimed fire we mean that the enemy's weapons are being fired into the friendly force's area without regard to target locations. In other words, the weapons are not being aimed at any specific targets.

The basic assumptions made in this section provide a starting point for structuring our problem of estimating casualties for each MOS and rank in a given scenario. We will now discuss the fundamental ideas which underlie how we propose to further structure the problem.

Consider an arbitrary scenario. If all individuals of each rank and MOS were exposed to the same risks on the battlefield, then all individuals would have the same chance of becoming a casualty. It is most likely that this would not be the case, and the basic idea behind our algorithm is to focus on those factors which cause differences in the chance of survival for each MOS and rank.

The first factor which we consider is location on the battlefield. Some individuals would be located near the forward edge of the battle area (FEBA) where the hazards to their survival would probably be greater than at some distance from the FEBA. Therefore, our algorithm considers the hazards, or risks, associated with different areas of the battlefield.

An individual's chance of survival also depends upon the length of time he is exposed to the hazards and the firing rates of the enemy's weapons. Clearly, the chance for survival decreases as exposure time and firing rates increase. For a given scenario we have a fixed time period, so the time factor will not be explicitly addressed. However, we will consider the effect firing rates have on survival probability.

Another factor which we will consider is the difference in vulnerability to the enemy's weapons. We can illustrate this point with a simple example. Suppose a tank is operating in an area with an infantryman, and artillery is being fired into that area. Clearly, the infantryman is more vulnerable to an artillery round than is a crewman in the tank. Thus, the infantryman's chance for survival is less.

To briefly summarize, our solution approach will be to isolate, for individuals of each rank and MOS, three factors that influence their chance for survival. Those factors are:

1) location on the battlefield,
2) the firing rates of the enemy's weapons, and

3) vulnerability to the enemy's weapons.

The next two sections of this chapter deal with items 1 and 2, respectively, and Chapter III addresses vulnerability.

B. DIVIDING THE FRIENDLY FORCE'S AREA INTO ZONES

The preceding section discussed the basic assumptions of the model and the factors considered to be fundamental to an individual's survival. The first factor we will discuss is location on the battlefield, and we begin by considering a snapshot of the battlefield for our scenario. Let us first view the situation as being static. The positions of all forces are known and constant, and we can, therefore, determine the location of the forward edge of the battle area (FEBA).

For a given scenario we know the maximum effective ranges of the enemy's weapons and approximately where those weapons will be located. Now consider only the area occupied by the friendly forces and the location of the enemy's weapons and their ranges. We can divide the friendly force's area into zones according to distance from the FEBA, such that the number of weapons that can be fired into each zone decreases as the zones are farther away from the FEBA. For instance, suppose that the enemy has three different types of weapons, that there is a large number of each of these weapons, and the maximum effective ranges and distances beyond the FEBA are as follows:

Weapon Type	Distance Beyond the FEBA (M)	Max. Range (M)
1	800	3,000
2	3,500	8,000
3	400	1,200 .

We see that for each type of weapon there is a maximum distance that it can be fired into the friendly force's area. For our example the distances are:

Weapon Type	Max. Distance (M)
1	2,200
2	4,500
3	800 .

Now we can divide the friendly force's area into four adjacent zones based on these maximum distances. We shall refer to the distances from the FEBA that define each zone as its zonal limits, and those weapons which can be fired into a zone as its zonal weapons. Doing so for our example provides the zonal limits and zonal weapons as depicted in Figure 2.1.

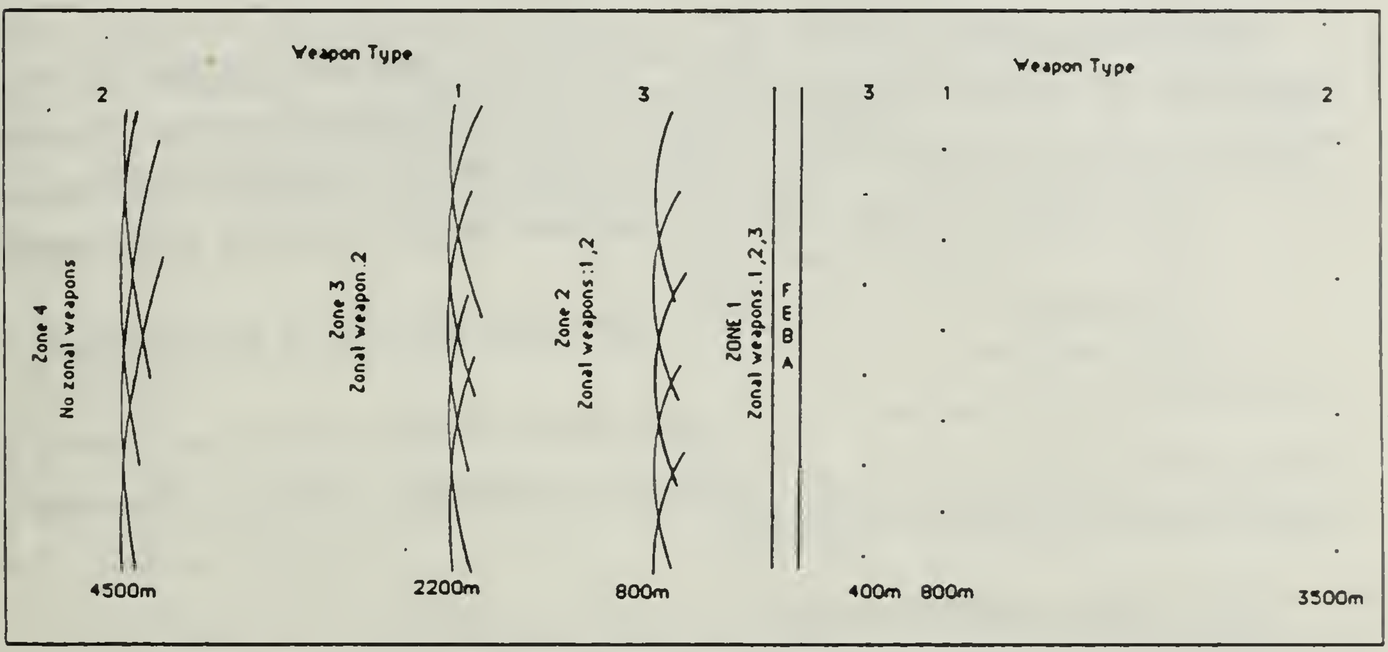

Figure 2.1 An example of zones in the friendly forces area.

We are now ready to consider the friendly forces in the static battlefield. Recall that, for a well defined scenario, we know how each MOS will be deployed within the friendly force's area. It is clear that all individuals of an MOS may or may not be located in the same zone and, therefore, may not be facing the same zonal weapons. Keep in mind that our casualty estimate is associated with some time period. For a short period of time, we can consider the zones stationary, and if we now allow the friendly forces positions to change during this length of time, the individuals of an MOS may move from one zone to another, then to a third zone, and so on. Each time an individual moves from one zone to another he faces a different potential threat, i.e., the zonal weapons. Therefore, to account for the effect of location on an individual's chance for survival we will assume that:

- we can estimate the proportion of the time period that an individual of each rank and MOS would spend in each zone, and
- individual positions are independent and uniformly distributed within each zone.

Let us briefly summarize what we have discussed in this section. Based upon the locations and ranges of the enemy's weapons we can divide the friendly force's area of the battlefield into adjacent zones, thus defining the zonal limits and zonal weapons. Given the time period, we can estimate how long an individual of each rank and MOS will be in each zone, and individual positions are uniformly distributed within the zones.

Dividing the friendly force's area into zones as described provides a way to account for the effect of battlefield location on an individual's survival. The zonal weapons represent a *potential* hazard to an occupant. We say *potential* for two reasons:

1) an occupant of a zone will only encounter hazards when the zonal weapons are, in fact, firing into the zone, and

2) an occupant of a zone cannot become a casualty if he is invulnerable to all zonal weapons.

The following section discusses the first point above. That is, how we can capture the effect of the firing rates on an individual's chance for survival. The topic of vulnerability is addressed in Chapter III.

C. THE ZONAL FIRING RATES

The previous section described how the friendly forces area can be divided into zones such that within each zone there is a different *potential* hazard. Clearly, an individual's probability of surviving within a zone is affected by the firing rates of the zonal weapons. In this section we consider how we will deal with the firing rates in our model.

It is reasonable to expect the military planners who are developing a contingency plan to have some concept of how intense the combat would be for their scenario. Thus, for a given scenario, the combat intensity, as measured by the firing rates of the enemy's weapons, would be known (or at least could be estimated). This implies that we can assign (estimated) values to the total firing rate of each type of enemy weapon. It is also reasonable to expect the planners to be familiar with the enemy's doctrine concerning target lists and priorities, and the coordination and use of supporting arms. Therefore, the planners should be able to further estimate the proportion of total rounds from each type of weapon that will be fired into each zone. It follows that we can estimate the rate at which each weapon would be firing into each zone. We shall call these rates the zonal firing rates.

We now make two important assumptions:

1) the arrival of impacts from each weapon in each zone constitute a Poisson process, and

2) the zonal firing rates are constant during the time period.

We are assuming constant zonal firing rates. However, in reality, presuming the enemy will suffer losses and that there are no enemy reinforcements during the time period, the zonal firing rate of a weapon will actually decrease over time. Suppose a zonal firing rate for one type of weapon would actually be as shown in Figure 2.2. Since the firing rate is changing over time, the arrivals of impacts in the zone actually constitute a non-homogeneous Poisson process. Let $\mathbf{I}(t)$ be the firing rate at time t. For a non-homogeneous Poisson process, the mean number of arrivals in the time interval (t_1,t_2) is

$$\int_{t_1}^{t_2} \mathbf{I}(t)\,dt,$$

or the area under the curve $\mathbf{I}(t)$ between t_1 and t_2 [Ref. 1: p.221]. We denote this area as $\mathbf{B}$, as shown in Figure 2.2.

Let $\mathbf{X}(t_1,t_2)$ be the number of rounds to impact in the zone in the time interval (t_1,t_2). Suppose the rounds are being fired by a weapon whose firing rate is as shown in Figure 2.2. Then $\mathbf{X}(t_1,t_2)$ is a Poisson distributed random variable with mean $\mathbf{B}$ [Ref..1: p.221].

For the same time interval (t_1,t_2) we can find a constant firing rate, λ, such that

$$\mathbf{B} = \lambda\,(t_2 - t_1) = \int_{t_1}^{t_2} \mathbf{I}(t)\,dt.$$

Thus, the probability distribution of $\mathbf{X}(t_1,t_2)$ is the same regardless of how $\mathbf{B}$ is computed. If the function $\mathbf{I}(t)$ is known it should be used to compute $\mathbf{B}$. However, if it is unknown we can estimate, for each weapon, a constant λ which we call the zonal firing rate.

If the zonal firing rate of Figure 2.2 were increased for the time period (t_1,t_2), $\mathbf{B}$ would increase and the probability distribution of $\mathbf{X}(t_1,t_2)$ would be more skewed toward higher values. Clearly, an individual's chance for survival within a zone decreases as the zonal firing rate increases.

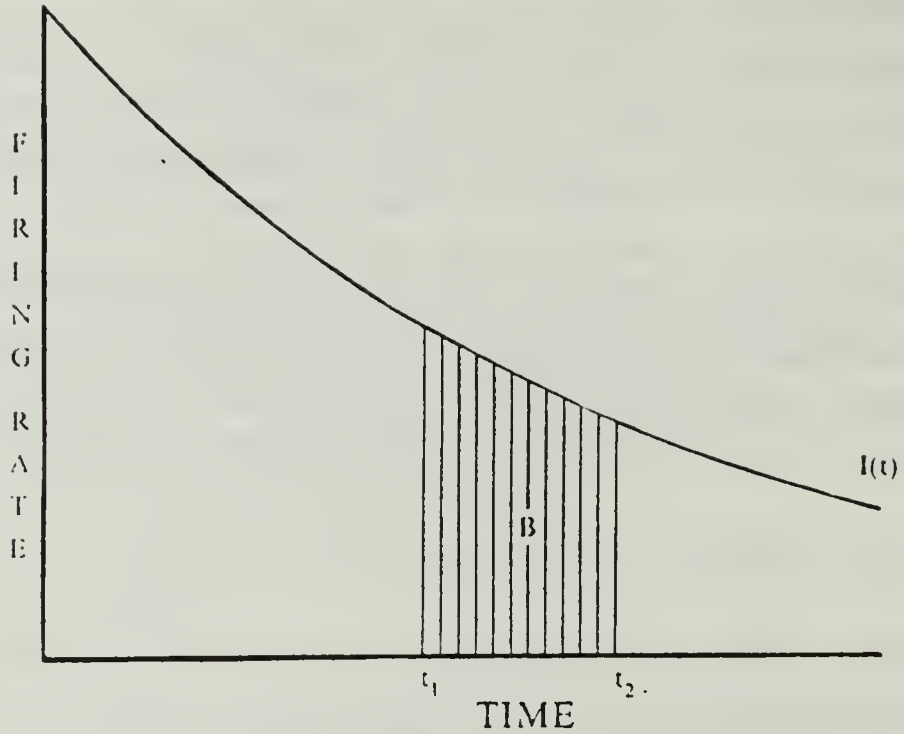

Figure 2.2 Computing the mean number of impacts during (t_1,t_2) for a weapon with a decreasing firing rate.

Again we will briefly summarize where we stand with our problem. We have a well defined scenario and for a specified time in conflict, we wish to estimate casualties for each MOS and rank. We can establish zones in the friendly force's area and we know which weapons can be fired into each zone. We assume that the enemy's fire is unaimed, that we can estimate the zonal firing rates which are constant for the time period, and that the impacts are independent and uniformly distributed within each zone. We know the proportion of the time period each rank and MOS spends in each zone, and individual positions are independent and uniformly distributed within each zone.

In this chapter we have attempted to structure the problem. We discussed what is assumed to be known about the problem, we defined some terms, and we discussed two of the three main ideas underlying our algorithm. Specifically, we discussed the effect on survival of:

1) the location on the battlefield, and
2) the firing rates of the enemy's weapons.

The third factor which fundamentally affects an individual's chance for survival is his vulnerability to the enemy's weapons. By vulnerability we mean the probability that

an individual will become a casualty, given his distance from a weapon's impact point, i.e., a miss distance. Thus, we define vulnerability as a single-shot conditional probability of kill. We are now ready to consider an individual's vulnerability and his chance for survival and derive our equation for estimating casualties. We do so in Chapter III.

III. DERIVING AN EQUATION FOR THE CASUALTY ESTIMATES

In the first section of this chapter we will discuss the third factor we consider basic to an individual's chance for survival on the battlefield; his vulnerability to the enemy's weapons. However, as our earlier definition implied, we will in fact be discussing conditional kill probabilities. Then, in the second section of this chapter, we will combine the effects of the three fundamental factors to find the probability that an individual will survive a period of time on the battlefield. In the final section we will derive the basic equation we use to estimate the number of casualties for each rank and MOS for the example in Chapter IV. Finally, in Chapter V we discuss several enhancements to the model.

For the remainder of our discussion we will suppose that the friendly forces are composed of m MOS's and r ranks, and that the enemy is firing w weapons into z zones. We shall use the following indices:

h = rank; h = 1,2,3...r,

i = MOS; i = 1,2,3...m,

j = weapon; j = 1,2,3...w,

k = zone; k = 1,2,3...z.

A. VULNERABILITY

Before we can find an individual's probability for survival, we must be able to characterize his vulnerability to the enemy's weapons. In other words, we must model the probability of kill conditioned on the miss distance, in accordance with our previous definition of vulnerability. Suppose we consider the effects of a weapon to be restricted to a circular area centered on its impact point. Suppose also that a target within this area becomes a casualty with probability 1.0, while a target outside of this area survives with probability 1.0. A weapon whose effects are modeled in this manner has been termed a "cookie cutter" weapon [Ref. 2: p.15-6]. We will consider the enemy's weapons as cookie cutter weapons.

For a cookie cutter weapon

$$P(k|d) = \begin{cases} 1 \text{ when } d \leq LR \\ 0 \text{ when } d > LR , \end{cases}$$

where d is the miss distance, and LR, the lethal radius, is a constant. We see that an individual either becomes a casualty, i.e., $d \leq LR$, or he does not, i.e., $d > LR$. We will assume that the lethal radius is the same for all ranks of an MOS, and that for each MOS i and weapon j the lethal radius, $(LR)_{ij}$, is known. Clearly, any difference in vulnerability to the enemy's weapons will be reflected by different lethal radii.

We have now discussed the third factor underlying our algorithm: an individual's vulnerability to the enemy's weapons. In the next section we will combine the three fundamental factors to determine an individual's chance for survival, and, in the final section of this chapter, we will derive our equation for estimating the number of casualties for each rank and MOS.

The important points to keep in mind are:

- personnel positions and weapon impacts are independent and uniformly distributed within each of the z zones,
- the arrival of weapon impacts constitute independent Poisson processes with rates λ_{jk}, and
- for each MOS i and weapon j the lethal radius, $(LR)_{ij}$, is known and applies to all ranks h.

B. THE PROBABILITY OF SURVIVAL

In this section we wish to determine an individual's probability of surviving some time period on the battlefield. In order to do so we must first consider the single-shot unconditional probability of kill for each MOS and weapon. This will lead us to deriving our equation for the casualty estimates in the following section.

For a cookie cutter weapon, the unconditional probability of kill, P(k), is

$$P(k) = P(k|d)\,P(d \leq LR) = (1)\,P(d \leq LR).$$

Given that an individual's position is uniformly distributed within a zone, $P(d \leq LR)$ is equal to the probability that an individual will be located within a circle of radius LR which is centered on an impact point [Ref. 2: p.15-8]. It follows that

$$P(k) = P(d \leq LR) = \pi\,(LR)^2 / A_k$$

where A_k is the area of zone k. Thus, we know the single-shot probability that an individual of MOS i will become a casualty in zone k due to weapon j. It is

$$P(k)_{ijk} = \frac{\pi\,(LR)_{ij}^{\,2}}{A_k}\,. \tag{3.1}$$

For an individual in a zone into which one weapon is firing at a rate λ_{jk}, the arrival of impacts constitute a Poisson process with rate λ_{jk}. Suppose there is a probability, $P(k)_{ijk}$, that each shot will cause the individual to become a casualty. By the thinning property of the Poisson process, rounds that will cause the individual to become a casualty constitute a Poisson process with rate $P(k)_{ijk}\lambda_{jk}$ [Ref. 1: p.204].

This holds for each of the w weapons. Thus, there are w independent Poisson processes. The rates of independent Poisson processes are additive, and the total arrival rate of casualty-producing rounds for MOS i in zone k is $R_{ik} = \Sigma_j P(k)_{ijk} \lambda_{jk}$ [Ref. 2: p.5-9]. If $X_{ik}(T)$ is the number of impacts that occur during a time period of length T in zone k, and those impacts cause casualties to MOS i, then $X_{ik}(T)$ is a Poisson distributed random variable with mean TR_{ik}. Therefore, we can write:

P { individual of MOS i survives a time period of length T in zone k }
$= P\{ X_{ik}(T) = 0 \}$
$= \exp\{ -T R_{ik} \}$
$= \exp\{ -T \Sigma_j P(k)_{ijk} \lambda_{jk} \}$.

For example suppose there are four weapons, and the firing rates into zone 2 are as follows:

$\lambda_{12} = 6$ /hr,
$\lambda_{22} = 8$ /hr,
$\lambda_{32} = 0$ /hr,
$\lambda_{42} = 3$ /hr.

Further suppose an individual of MOS 1 occupies the zone for two hours (T = 2.0), and the associated single-shot kill probabilities are:

$P(k)_{112} = 0.063$,
$P(k)_{122} = 0.042$,
$P(k)_{132} = 0.085$,
$P(k)_{142} = 0.107$.

There are four independent Poisson processes involved, one for each weapon. The total arrival rate of impacts in zone 2, that will cause the individual to become a casualty, is the sum of the rates of the four independent Poisson processes. It is

$R_{12} = \Sigma_j P(k)_{ijk} \lambda_{jk}$,
$R_{12} = \{ (0.063)(6) + (0.042)(8) + (0.085)(0) + (0.107)(3) \}$ / hr,
$R_{12} = 1.035$ / hr.

After a time period of two hours:

$$
\begin{aligned}
&P\{\text{ individual of MOS 1 survives two hours in zone 2 }\}\\
&= P\{X_{12}(2) = 0\}\\
&= \exp\{-(2)(1.035)\}\\
&= 0.126.
\end{aligned}
$$

From the scenario we can estimate the proportion of the time in conflict that an individual of MOS i and rank h will be in each zone. We shall denote these proportions as α_{ihk} and note that $\Sigma_k \alpha_{ihk} = 1.0$, for all i and h, since each individual must be within the friendly force's area for the entire time period. Additionally, it is noted that α_{ihk} is also the fixed proportion of MOS i and rank h that would occupy zone k during the time period. In order for an individual of MOS i and rank h to survive a time period of length T, he must survive in each zone k for the length of time $T\alpha_{ihk}$. Thus,

$$P\{\text{ individual of MOS i and rank h survives a time period of length T }\}$$

$$= \Pi_k \; P\{\text{ individual of MOS i and rank h survives in zone k for a time } T\alpha_{ihk}\}$$

$$= \Pi_k \; \exp\{-T\,\alpha_{ihk}\,R_{ik}\}$$

$$= \exp\{-T\;\Sigma_k \alpha_{ihk}\;R_{ik}\}$$

$$= \exp\{-T\,\Sigma_k \alpha_{ihk}\,\Sigma_j\,P(k)_{ijk}\,\lambda_{jk}\}. \tag{3.2}$$

Now that we have an equation to calculate the probability of survival for an individual of MOS i and rank h, we can proceed to find an equation that will yield the casualty estimates.

C. THE CASUALTY ESTIMATES

Equation 3.2 is the expression for an individual's probability of surviving a length of time T on the battlefield. The complement of this probability is

$$P\{\text{ individual of MOS i and rank h becomes a casualty during the time period T }\}$$

$$= 1 - \exp\{-T\;\Sigma_k \alpha_{ihk}\,\Sigma_j\,P(k)_{ijk}\,\lambda_{jk}\}.$$

This probability can also be estimated by

$$C_{ih}(T) \,/\, N_{ih}\,,$$

where $C_{ih}(T)$ is the number of MOS i, rank h casualties at time T, and N_{ih} is the initial strength of MOS i and rank h. Thus,

$$C_{ih}(T) / N_{ih} = P\{\text{ individual of MOS i and rank h becomes a casualty during the time period } T\},$$

$$C_{ih}(T) = N_{ih}\{1 - \exp(-T\ \Sigma_k \alpha_{ihk}\ \Sigma_j\ P(k)_{ijk}\ \lambda_{jk})\},$$

for all i, j, k, and h. Now substituting for $P(k)_{ijk}$ from Equation 3.1 yields

$$C_{ih}(T) = N_{ih}\{1\text{-}\exp(-T\pi\Sigma_k\alpha_{ihk}\Sigma_j\frac{(LR)_{ij}^2}{A_k}\lambda_{jk})\}. \tag{3.3}$$

This is the equation we will use to estimate the number of casualties to MOS i and rank h. Note that all N_{ih}, α_{ihk}, λ_{jk}, and $(LR)_{ij}$ are known, or can be estimated from the scenario. Thus, for a given time in conflict T, the casualty estimates can be computed directly from Equation 3.3.

The casualty estimates that would result from Equation 3.3 reflect the three factors we consider fundamental to an individual's survival for the time period T. Location on the battlefield is taken into account by the α_{ihk}. They reflect the amount of time that an individual of MOS i and rank h, or the fixed proportion of MOS i and rank h that, is exposed to the *potential* hazards of zone k. The firing rates, λ_{jk}, clearly affect the casualty estimates, and vulnerability is represented by the lethal radii and zonal areas. This section concludes the development of the algorithm. Chapter IV contains a worked example for a hypothetical scenario.

IV. EXAMPLE

This chapter provides an example of using our model to predict the casualties for a hypothetical scenario. Although the situation is very simple and the values chosen for the variables may not be realistic, the example will, nonetheless, illustrate the use of the model. Some of the information required by the model would be known for our scenario and some must be estimated. That distinction will be made clear throughout the example.

For our hypothetical scenario we know the type, number, and ranges of the enemy's weapons, and we know the doctrinal placement of those weapons on the battlefield. Suppose the enemy's arsenal is composed of five types of weapons, and the distance beyond the FEBA and the maximum range for each weapon is:

Weapon Type	Distance Beyond the FEBA (M)	Max. Range (M)
1	1,500	6,700
2	800	6,000
3	2,000	4,500
4	1,000	3,500
5	700	1,500

We see that the maximum distance into the friendly force's area that each weapon can be fired is:

Weapon Type	Max. Distance (M)
1	5,200
2	5,200
3	2,500
4	2,500
5	800

We can now establish the zonal limits and determine the zonal weapons. They are:

	Zonal Limits (m)	Zonal Weapons
Zone 1	FEBA - 800	1,2,3,4,5,
Zone 2	800 - 2500	1,2,3,4,
Zone 3	2500 - 5200	1,2.

We must now estimate the zonal firing rates. Suppose the total firing rate of each type of weapon has been estimated by the method discussed in Chapter II, Section C, and that we have estimated the proportion of rounds from each weapon to be fired into each zone. We can then estimate the zonal firing rates as follows:

		Est. Firing Rate (/hr)	Est. Proportion Into Zone			Est. Firing Rate Into Zone		
			1	2	3	1	2	3
W								
E	1	20	0.60	0.30	0.10	12.00	6.00	2.00
A	2	35	0.15	0.25	0.60	5.25	8.75	21.00
P	3	50	0.45	0.55	0.00	22.50	27.50	0.00
O	4	40	0.30	0.70	0.00	12.00	28.00	0.00
N	5	60	1.00	0.00	0.00	60.00	0.00	0.00 .

This is all of the information we need concerning the enemy. We will now turn our attention to the friendly forces in our scenario. Suppose there are four MOS's, that each MOS has within it the same three ranks, and the initial strengths are as follows:

		MOS 1	MOS 2	MOS 3	MOS 4
R					
A	1	60	50	135	35
N	2	100	75	75	35
K	3	40	125	90	30 .

We know how the friendly forces would be deployed on the battlefield, and we can determine the width of the friendly force's area. If the friendly forces would be displaced along a five hundred meter front, the zonal areas are:

	Zonal limits (m)	Area (km^2)	Area (m^2)
Zone 1	FEBA - 800	0.40	4.0×10^5
Zone 2	800 - 2500	0.85	8.5×10^5
Zone 3	2500 - 5200	1.35	1.35×10^6 .

We have examined the missions of each rank and MOS and estimated the proportion of a time period that individuals of each rank and MOS would spend in each zone. The estimates are:

		MOS 1			MOS 2			MOS 3			MOS 4		
		Zone			Zone			Zone			Zone		
R		1	2	3	1	2	3	1	2	3	1	2	3
A	1	.6	.2	.2	.2	.7	.1	.4	.1	.5	.0	.8	.2
N	2	.5	.2	.3	.1	.6	.3	.2	.3	.5	.0	.5	.5
K	3	.1	.1	.8	.0	.8	.2	.3	.3	.4	.0	.3	.7 .

The last data we need are the lethal radii. Suppose they are (in meters):

		MOS 1	MOS 2	MOS 3	MOS 4
W					
E	1	27	35	10	16
A	2	35	45	13	21
P	3	15	20	6	9
O	4	0	15	5	7
N	5	40	53	16	24 .

We now have all the information we need to compute the casualty estimates. Our equation for estimating the number of casualties for MOS i and rank h during a time period of length T is

$$C_{ih}(T) = N_{ih} \{ 1 - \exp(-T \pi \Sigma_k \alpha_{ihk} (1/A_k) \Sigma_j (LR)_{ij}^2 \lambda_{jk}) \}, \tag{4.1}$$

for i = 1,2,3,4 and h = 1,2,3. We will work through the calculation for MOS 1 and rank 1 and then provide the results for the others. For MOS 1 and rank 1:

$i = 1$	$\alpha_{111} = 0.6$	$A_1 = 4.0 \times 10^5 \ m^2$	$(LR)_{11} = 27$ m
$h = 1$	$\alpha_{112} = 0.2$	$A_2 = 8.5 \times 10^5 \ m^2$	$(LR)_{12} = 35$ m
$N_{11} = 60$	$\alpha_{113} = 0.2$	$A_3 = 1.35 \times 10^6 \ m^2$	$(LR)_{13} = 15$ m
			$(LR)_{14} = 0$ m
			$(LR)_{15} = 40$ m .

The estimated firing rates were given earlier, but we will now list them explicitly. They are:

$\lambda_{11} = 12.00$	$\lambda_{12} = 6.00$	$\lambda_{13} = 2.00$
$\lambda_{21} = 5.25$	$\lambda_{22} = 8.75$	$\lambda_{23} = 21.00$
$\lambda_{31} = 22.50$	$\lambda_{32} = 27.50$	$\lambda_{33} = 0.00$
$\lambda_{41} = 12.00$	$\lambda_{42} = 28.00$	$\lambda_{43} = 0.00$
$\lambda_{51} = 60.00$	$\lambda_{52} = 0.00$	$\lambda_{53} = 0.00$

It would be cumbersome to write out Equation 4.1 completely. Therefore, we will compute the argument of the exponential first. Substituting the appropriate values into

$$\alpha_{ihk} \{1 / A_k \} \Sigma_j (LR)_{ij}^2 \ \lambda_{jk}$$

we have:

for zone 1,

k = 1,

$$\alpha_{111} \{ 1 / A_1 \} \Sigma_j (LR)_{1j}^2 \ \lambda_{j1}$$

$$= \frac{0.6}{4.0\mathrm{x}10^5} \ \{(27)^2(12)+(35)^2(5.25)+(15)^2(22.50)+(0)^2(12.00)+(40)^2(60.00)\}$$

$$= 0.1744;$$

for zone 2,

k = 2,

$$\alpha_{112} \{ 1 / A_2 \} \Sigma_j (LR)_{1j}^2 \ \lambda_{j2}$$

$$= \frac{0.2}{8.5\mathrm{x}10^5} \ \{(27)^2(6.0)+(35)^2(8.75)+(15)^2(27.50)+(0)^2(28.00)+(40)^2(0)\}$$

$$= 0.0050;$$

and for zone 3,

$k = 3$,

$\alpha_{113} \{ 1 / A_3 \} \Sigma_j (LR)_{1j}^2 \ \lambda_{j3}$

$$= \frac{0.2}{1.35 \times 10^6} \{(27)^2(2.0) + (35)^2(21.0) + (15)^2(0) + (0)^2(0) + (40)^2(0)\}$$

$= 0.0040.$

Summing over the zones, k, yields:

$\Sigma_k \alpha_{11k} \{ 1 / A_k \} \Sigma_j (LR)_{1j}^2 \ \lambda_{jk}$

$= 0.1744 + 0.0050 + 0.0040$

$= 0.1834.$

Now substituting into Equation 4.1 we have:

$C_{11}(T) = N_{11} \{ 1 - \exp(-T \pi \ \times \ 0.1834) \},$

$C_{11}(T) = 60 \{ 1 - \exp(-T \pi \ \times \ 0.1834) \}.$

If we want to estimate the casualties for a four hour time period, then:

$C_{11}(4) = 60 \{ 1 - \exp(-4 \pi \ \times \ 0.1834) \},$

$C_{11}(4) = 54.01,$

which could be rounded to 54. We estimate the other casualties, rounded to the nearest integer, to be:

$C_{12}(4) = 86,$

$C_{13}(4) = 18,$

$C_{21}(4) = 42,$

$C_{22}(4) = 51,$

$C_{23}(4) = 55,$

$C_{31}(4) = 31,$

$C_{32}(4) = 10,$

$C_{33}(4) = 17,$

$C_{41}(4) = 4,$

$C_{42}(4) = 4,$

$C_{43}(4) = 3.$

As a percentage of initial strength the casualty estimates range from a high of 90.0% for MOS 1 and rank 1, to a low of 10.0% for MOS 4 and rank 3. This large difference can be explained by contrasting the input data that reflect the three factors we consider fundamental to an individual's chance for survival. For this scenario, individuals of MOS 4 and rank 3 would spend seventy percent of a time period in zone 3 where there are only two zonal weapons, whereas individuals of MOS 1 and rank 1 would spend sixty percent of a time period in zone 1 with five zonal weapons. Thus, location on the battlefield causes greater *potential* risks for individuals of MOS 1 and rank 1.

We estimated the zonal firing rates as follows:

WEAPON	Est. Firing Rate (/hr)	Est. Proportion Into Zone 1	2	3	Est. Firing Rate Into Zone 1	2	3
1	20	0.60	0.30	0.10	12.00	6.00	2.00
2	35	0.15	0.25	0.60	5.25	8.75	21.00
3	50	0.45	0.55	0.00	22.50	27.50	0.00
4	40	0.30	0.70	0.00	12.00	28.00	0.00
5	60	1.00	0.00	0.00	60.00	0.00	0.00

Suppose they had been estimated to be:

WEAPON	Est. Firing Rate (/hr)	Est. Proportion Into Zone 1	2	3	Est. Firing Rate Into Zone 1	2	3
1	20	0.70	0.30	0.00	14.00	6.00	0.00
2	35	0.75	0.25	0.00	26.25	8.75	0.00
3	50	0.45	0.55	0.00	22.50	27.50	0.00
4	40	0.30	0.70	0.00	12.00	28.00	0.00
5	60	1.00	0.00	0.00	60.00	0.00	0.00

We see that compared to the former estimates two of the zonal firing rates for zone 3 have decreased and those for zone 1 have increased. It follows that the risks to

occupants of zone 1 have increased, and for zone 3 the risks have decreased. We would expect the casualty estimate for MOS 1 and rank 1 to increase from the former value of 54 and the casualty estimate for MOS 4 and rank 3 to decrease from 3. In fact, using the latter set of firing rates provides the following estimates:

$C_{11}(4)$ = 56, which is 92.8% of initial strength,
$C_{43}(4)$ = 1, which is 3.8% of initial strength .

Individuals of MOS 4 are less vulnerable to all but one of the enemy's weapons than are individuals of MOS 1. The difference in vulnerability is reflected by the different lethal radii. If we exchanged the lethal radii between MOS 1 and MOS 4, with all other data used for the original casualty estimates held constant, the estimates would be:

$C_{11}(4)$ = 34, which is 56.7% of initial strength, and
$C_{43}(4)$ = 7, which is 23.3% of initial strength.

We see that by increasing the lethal radii for MOS 4 the casualty estimate $C_{43}(4)$ rose from 3 to 7. $C_{11}(4)$ decreased from 54 to 34 due to a decrease in the lethal radii for MOS 1.

The example of this chapter illustrates the use of the model for a simple, hypothetical scenario. The model is not complex, it does not require a large amount of input, and the casualty estimates are easily computed. In the next and final chapter of this thesis we will discuss how the model can be enhanced for added realism.

V. ENHANCING THE MODEL'S REALISM AND USEFULNESS

In Chapters II and III we developed an algorithm to forecast casualties, and Chapter IV provided an example of using the algorithm with a simple scenario. The algorithm focuses on three factors which are considered fundamental to an individual's survival on the battlefield. Those factors are:

1) an individual's location on the battlefield,
2) the firing rates of the enemy's weapons, and
3) an individual's vulnerability to the enemy's weapons.

To isolate the effects of these factors on the probability of survival for individuals of each rank and MOS, our model requires some input data and several assumptions. The first two factors listed above are taken into account by input data. Specifically, we discussed the fact that the planners would have to provide two sets of data. One set is the proportion of a time period that an individual of each rank and MOS would spend in each zone (the α_{ihk}), and the second set is the zonal firing rates (the λ_{jk}). To account for the effect of the third factor above, we defined vulnerability as the probability of kill conditioned on the miss distance d. We were then able to compute the unconditional, single-shot kill probabilities, $P(k)_{ijk}$, by assuming that:

1) all of the enemy's fire is unaimed,
2) the effects of the enemy's weapons can be adequately modeled with the cookie-cutter concept, and
3) individual positions and weapon impacts are uniformly distributed within each zone.

In this chapter we will consider how the model can be enhanced to provide added realism in two ways. First, we will discuss situations where the above mentioned assumptions would not hold. We will see that our algorithm is flexible enough to accommodate assumptions other than those we made to compute the $P(k)_{ijk}$. Secondly, we will discuss how values for the α_{ihk} and the λ_{jk} might be determined. Then in the third section of this chapter we will make a few remarks to conclude this thesis.

A. USING THE ALGORITHM WITH VARYING ASSUMPTIONS

The assumptions listed above lead to the $P(k)_{ijk}$ as described by this model. Therefore, if any of these assumptions do not hold, the formula we derived for

computing the $P(k)_{ijk}$ values may not be appropriate. Hence, the discussion that follows actually concerns computation of the $P(k)_{ijk}$ under different assumptions.

We will first consider our assumption that all of the enemy's fire is unaimed. Clearly, this assumption would not hold when the enemy can select specific targets for his weapons, and our model can be enhanced by including aimed fire in this case. There are two types of weapons that would have to be considered for aimed fire:

- weapons that would be fired only at specific targets, and
- weapons that could be fired in either an aimed or unaimed mode.

An anti-tank missile, which would only be used against armored targets, is an example of the first type of weapon. Artillery is an example of the second type since it could be fired unaimed into a zone for harassment or interdiction purposes, or it could be directed against specific targets.

For a weapon such as artillery, which can be aimed or unaimed, the planners would have to estimate how the enemy would divide its use between the two modes of fire. Some of the fires from those types of weapons could be considered aimed and the remaining fires unaimed. We would then have two distinct sets of weapons, i.e., the aimed and the unaimed fire weapons. The effect of each set of weapons on an individual's chance for survival could be handled separately.

All of the unaimed fire could be modeled exactly as discussed in this thesis. To incorporate aimed fire we would need to add a third subscript, i, to the zonal firing rates. Thus, λ_{ijk} would indicate the estimated rate at which the enemy fires weapon j at MOS i in zone k. For aimed *and* unaimed fire,

$$\begin{aligned} &P(\text{individual of MOS i survives a time period of length T in zone k}) \\ &= \exp \{ -T(\Sigma_{ufw} P(k)_{ijk}\, \lambda_{jk} + \Sigma_{afw}\, P(k)_{ijk}\, \lambda_{ijk}) \}, \end{aligned} \tag{5.1}$$

where

ufw is the set of unaimed fire weapons, and
afw is the set of aimed fire weapons.

However, for aimed fire the $P(k)_{ijk}$ depends upon the firing errors of weapon j, and those errors would have to be known to compute the above probability of survival. Thus, if the firing errors were known for the set of aimed fire weapons and all λ_{ijk} could be estimated, aimed fire could be easily incorporated into our model.

We can see from Equation 5.1 that an individual's chance for surviving a time period of length T depends upon the firing rates and the probabilities of kill. For our model we computed the single-shot kill probabilities assuming that a weapon's effects, or damage potential, could be realistically modeled with the cookie-cutter concept. If there is a more appropriate model for the $P(k|d)$, it could be readily included in our casualty stratification model through the calculation of the $P(k)_{ijk}$ values. For an unaimed fire weapon, the unconditional single-shot kill probability could be computed knowing the function for $P(k|d)$ and the probability density function for d, the miss distance. For aimed fire weapons, finding the $P(k)_{ijk}$ would also require knowing the firing errors as mentioned above.

In order to compute the $P(k)_{ijk}$ we also assumed that an individual's position and all weapon impacts are uniformly distributed within each zone. These assumptions would not be required for the targets and impacts of an aimed fire weapon, and the $P(k)_{ijk}$ could be computed as previously described. However, to compute the $P(k)_{ijk}$ for an unaimed fire weapon, *some* distribution would have to be specified for both an individual's position and the weapon impacts. If these distributions were more appropriate or realistic than the uniform distributions we assumed, they also could be incorporated in the model through the calculations of the probabilities of kill.

We can summarize this section by stating that different assumptions concerning a weapon's damage ability, the firing errors of an aimed weapon, the impacts of a unaimed weapon, or an individual's location can be accommodated by our model through the calculation of the $P(k)_{ijk}$ in Equation 5.1. Depending upon the functions for $P(k|d)$ and the probability density functions for the firing errors of the aimed weapons, the impacts of the unaimed weapons, and an individual's position, there may be closed form solutions for $P(k)_{ijk}$ or they may have to be determined numerically or by a Monte Carlo technique.

B. MODEL DATA NEEDS

We have seen that our model requires specific data to compute the casualty estimates, and in this section we will consider the data that must be supplied by the planners. These data are the proportion of a time period that individuals of each MOS and rank would spend in each zone (the α_{ihk}) and the zonal firing rates (the λ_{jk}). Obviously, the data can be obtained by simple, subjective estimation. However, for each set of data we will discuss a method for obtaining somewhat more objective and realistic values. We will first discuss finding values for the zonal firing rates and then the time proportions.

If the function for the firing rate of a weapon over time, **I**(t), is known, finding the constant λ would be quite easy. However, if **I**(t) is not known, it might be difficult to properly estimate the firing rates. Clearly, the firing rates might not be known for a scenario, but perhaps a separate model could be developed to estimate the decrease in the enemy's firing rates over the time of the battle. There are models, readily available, to do just that, but they require the additional input of attrition coefficients. However, the usefulness of our casualty stratification model would be greatly enhanced by a method to estimate the function **I**(t) for each type of weapon in the enemy's arsenal.

There are other realistic situations which can be taken into account by the values assigned to the firing rates in our model. For instance, the firing rate of a weapon could be limited by the enemy's ammunition resupply capabilities, or the firing rate of an aimed weapon could depend upon the enemy's ability to detect targets. These are matters which would have to be considered by the planners during the process of estimating the zonal firing rates, but the model requires no modification to accommodate these possibilities. However, there are situations which our model can not easily handle. For instance, it is likely that some of the firing rates might be dependent, and incorporating dependent firing rates would require structural change in the model.

We will now discuss estimating the α_{ihk}. In our model these proportions are actually parameters which account for the amount of time individuals of MOS i and rank h are exposed to the hazards in zone k. Thus, they reflect one of the three fundamental factors, i.e., location on the battlefield, and their estimation warrants some effort.

Values for the α_{ihk} can be obtained by soliciting expert opinion. One technique which can be used has been termed the constant sum method [Ref. 3]. This method uses a least squares approach to determine ratio scale values from the subjective inputs of judges. This method could be used with any number of judges and requires only modest computational effort, but the questionnaires would be lengthy in this situation. However, this is one method which could be used to provide somewhat more realistic values for the α_{ihk} thereby enhancing the usefulness of our model.

In this section we discussed how the data, which must be supplied by the planners, can be obtained by means other than simple, subjective estimation. The usefulness and realism of our casualty stratification model would be greatly enhanced by implementing the procedures mentioned. Unfortunately, however, the costs of the

enhancements discussed, in terms of obtaining the additional data required, could be rather high.

C. CONCLUSIONS

The algorithm and model developed in this thesis are conceptually simple. The model does not require a great deal of data, and the casualty estimates are easily computed. We have discussed how the algorithm can be enhanced for added realism in situations where our assumptions would not be valid, and we have seen that there are methods which can be used to obtain some of the data required by our model. It is sincerely hoped that the effort embodied in this thesis will be beneficial to those faced with the problem of forecasting battle casualties by MOS and rank.

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c.1 A casualty stratification model.